Magnetic field Energy Calculated by Magnetic Vector Potential in Open-Loop Problems

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In closed-loop problems, magnetic field energy might be calculated by many methods, including magnetic field method and magnetic vector potential method. But previous work showed that the two mentioned methods may not reach an agreement in open-loop problems. Therefore, this paper deduces calculative formula in open-loop problem and finds that the effect of displacement current is the key reason. The method of magnetic vector potential with displacement current is proposed. Analytical analysis and numerical experiment are given and show that validity of this method.

***Index Terms*—displacement current, magnetic field energy, magnetic vector potential, open-loop problem**

I. Introduction

OPEN loop problems that analysis in un-closed loops are common and practical because of unknown current return path. Ruehli's works [1-2] showed that open loop problems could be as well solved by both considering inductive and capacitive parts in PEEC model, which means that the open loop could also be closed capacitively and is a real physics based model.

Therefore, magnetic field energy could be used for calculation of the inductive part [3], as shown in (1).

$$W_{\mathrm{M}} = \frac{1}{2}LI^2 = \frac{1}{2}\int \boldsymbol{B}\cdot\boldsymbol{H}\mathrm{d}v = \frac{1}{2}\int \boldsymbol{A}\cdot\boldsymbol{J}\mathrm{d}v \tag{1}$$

In many cases, the last expression in (1) which involves magnetic vector potential $\boldsymbol{A}$ is often applied in calculation, where $\boldsymbol{J}$ is the conduction current in closed loop problem.

However, in open loop problems Kalhor's works [4-5] showed that surface charges accumulating with time need to be placed at both ends of a conductor segment and generate displacement current in space. In addition, Ni's works [6] illustrated that those two methods in (1) may obtain different results in open-loop problems without consideration of displacement current. So (1) might be transformed as follows in open-loop problems,

$$W_{\mathrm{M}} = \frac{1}{2}\int\left(\boldsymbol{A}_{\mathrm{C}}\cdot\boldsymbol{A}_{\mathrm{D}}\right)\left(\boldsymbol{J}_{\mathrm{C}}\cdot\boldsymbol{J}_{\mathrm{D}}\right)\mathrm{d}v \tag{2}$$

where subscript C and D represent conduction current and displacement current respectively.

This paper presents a method to calculate magnetic field energy by vector potential A in open-loop problems. Section II shows formulas of different parts in (2). In Section III, numerical experiment is made to compare the different parts in (2). Different methods shown in (1) are also compared. Results show that displacement current is a necessary part in open loop problems

II. Derivation of formulas

As illustrated in (2), there are four parts in the calculation of magnetic field energy. For a simple example in Fig 1, $\boldsymbol{J}_{\mathrm{C}}$ is conduction current in the conductor and uniformly distributed on cross section of the conductor

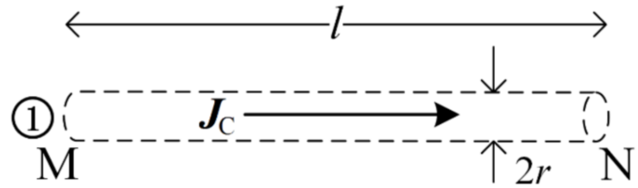

Fig. 1. A cylindrical conductor segment with length *l* and radius *r*.

$$\boldsymbol{J}_{\mathrm{C}}(\boldsymbol{r}) = \begin{cases}\boldsymbol{J}_{\mathrm{C}} & \text{inside conductor}\\ 0 & \text{outside conductor}\end{cases} \tag{3}$$

where $\boldsymbol{r}$ is space vector pointing to field point. $\boldsymbol{J}_{\mathrm{D}}$ is the displacement current generated by charges accumulating in the end surfaces M and N.

$$\boldsymbol{J}_{\mathrm{D}}(\boldsymbol{r}) = \frac{J_{\mathrm{C}}}{4\pi}\left(\int_{\mathrm{SN}}\frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SN}}}{\left|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SN}}\right|^3}\mathrm{d}s - \int_{\mathrm{SM}}\frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SM}}}{\left|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SM}}\right|^3}\mathrm{d}s\right) \tag{4}$$

where $\boldsymbol{r}_{\mathrm{SN}}$ and $\boldsymbol{r}_{\mathrm{SM}}$ are space vector pointing from surface M and N to field point. $\boldsymbol{A}_{\mathrm{C}}$ is magnetic vector potential generated by conduction current $\boldsymbol{J}_{\mathrm{C}}$.

$$\boldsymbol{A}_{\mathrm{C}}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int_{\mathrm{V}}\frac{\boldsymbol{J}_{\mathrm{C}}}{\left|\boldsymbol{r}-\boldsymbol{r}_v\right|}\mathrm{d}v \tag{5}$$

$\boldsymbol{A}_{\mathrm{D}}$ is magnetic vector potential generated by displacement current $\boldsymbol{J}_{\mathrm{D}}$ [6].

$$\boldsymbol{A}_{\mathrm{D}}(\boldsymbol{r}) = \frac{\mu_0 J_{\mathrm{C}}}{8\pi}\left(\int_{\mathrm{SN}}\frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SN}}}{\left|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SN}}\right|}\mathrm{d}s - \int_{\mathrm{SM}}\frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SM}}}{\left|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{SM}}\right|}\mathrm{d}s\right) \tag{6}$$

By combining (3)-(6), the value of (2) may be obtained.

$$\begin{aligned} W_{\mathrm{M}} &= W_{\mathrm{CC}} + W_{\mathrm{DC}} + W_{\mathrm{CD}} + W_{\mathrm{DD}} \\ &= \frac{1}{2}\int\left(\boldsymbol{A}_{\mathrm{C}}\cdot\boldsymbol{J}_{\mathrm{C}} + \boldsymbol{A}_{\mathrm{C}}\cdot\boldsymbol{J}_{\mathrm{D}} + \boldsymbol{A}_{\mathrm{D}}\cdot\boldsymbol{J}_{\mathrm{C}} + \boldsymbol{A}_{\mathrm{D}}\cdot\boldsymbol{J}_{\mathrm{D}}\right)\mathrm{d}v \end{aligned} \tag{7}$$

III. Numerical experiment

For the example in Fig 1, different methods in (1) are applied and compared in Fig 2, including the method of magnetic field and the method of magnetic vector potential.

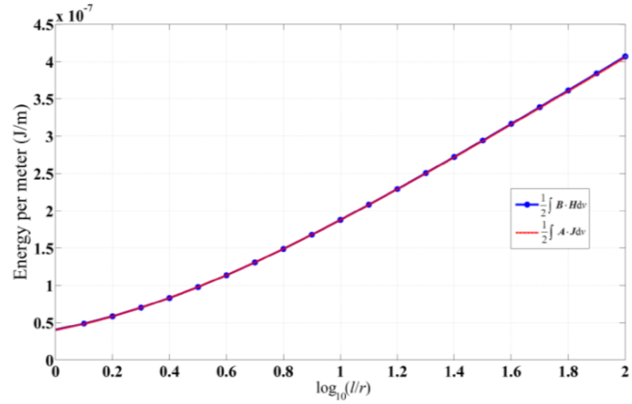

Fig. 2. Comparison of different methods for model in Fig 1.

where the horizontal axis is the ratio of length l to radius r, which means conductors with different shape. The vertical axis is the magnetic field energy per meter, which means the value of energy W divides the length l. FSV tool [7-8] is used for analyzing the agreement of these two curves in Fig 3

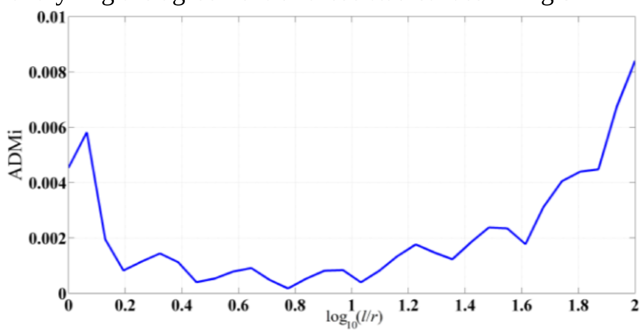

Fig. 3. Results of different parts in (7) for model in Fig 1.

According to evaluation of FSV method, the matching between these two methods is “excellent”(where ADMi<0.1), whereas the values of GRADE and SPREAD are both 1 under default threshold of 85%. Hence the proposed method is verified. Then each part of (7) is compared in Fig 4.

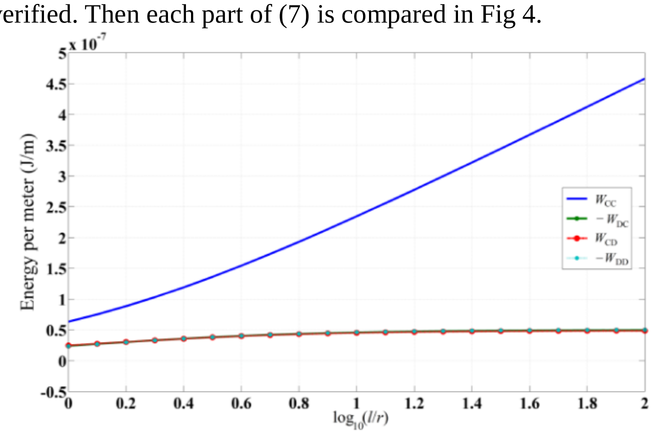

Fig. 4. Results of different parts in (7) for model in Fig 1.

Results show that absolute values of WCC, WDC and WDD are very close to each other. So the following relationship might be obtained

$$-W_{\mathrm{DC}} = W_{\mathrm{CD}} = -W_{\mathrm{DD}} \tag{8}$$

Combining (7) and (8)

$$W_{\mathrm{M}} = W_{\mathrm{CC}} + W_{\mathrm{DC}} + W_{\mathrm{CD}} + W_{\mathrm{DD}} = W_{\mathrm{CC}} + W_{\mathrm{DC}} \tag{9}$$

Therefore, the contribution of displacement current need to be considered in open-loop problems.

IV. Conclusion

This paper presents a method for calculation of the magnetic field energy in open-loop problems. According to the method in closed loop problems, the necessary of displacement current JD is discussed, which shows JD might obviously influence the results. The proposed method is compared with existing method and verified in numerical experiment.

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